MATHEMATICS-1(1st Semester Diploma Engg:-2024-25)
UNIT-III

02 Mark Questions & Solutions **Taxonomy Level**

1. Find the value of $(-\mathbf{i})^{\mathbf{4n+2}}$. Level-2(Understanding)

Solution:
$(-i)^{4n+2} = (-i)^{4n} \cdot (-i)^2 = ((-i)^4)^n \cdot (-i)^2 = i^{4^n} \cdot -1 = 1^n \cdot -1 = -1$

2. **What is the value of** $\mathbf{arg(\omega) + arg(\omega^2)}$? Level-2(Understanding)

Solution:

Since $\arg(z_1) + \arg(z_2) = \arg(z_1 . z_2)$.

$\Rightarrow \arg(\omega) + \arg(\omega^2) = \arg(\omega \times \omega^2) = \arg(\omega^3) = \arg(1) = 0.$

Therefore, $\arg(\omega) + \arg(\omega^2) = 0$

3. **Find the conjugate of** $\frac{\mathbf{1}}{\mathbf{3+4i}}$. Level-2(Understanding)

Solution:

$$\text{Let } z = \frac{1}{3+4i} = \frac{1(3-4i)}{(3+4i)(3-4i)} = \frac{3-4i}{3^2-(4i)^2} = \frac{3-4i}{9-16(i)^2}$$
$$= \frac{3-4i}{9-16(-1)} = \frac{3-4i}{9+16} = \frac{3-4i}{25} = \frac{3}{25} - \frac{4}{25}i$$

$$\text{So, } \bar{z} = \frac{3}{25} + \frac{4}{25}i$$

$$\text{Thus, conjugate of } \frac{1}{3+4i} = \frac{3}{25} + \frac{4}{25}i$$

4. Find the multiplicative inverse of $\mathbf{4 - 3i}$. Level-2(Understanding)

Solution:

$\bar{z} = 4 + 3i$ & $|z| = \sqrt{4^2 + 3^2} = \sqrt{25}$

$$z^{-1} = \frac{\bar{z}}{|z|^2} = \frac{4+3i}{25} = \frac{4}{25} + \frac{3}{25}i$$

Thus, the multiplicative inverse of $4 - 3i$ is $\frac{4}{25} + \frac{3}{25}i$

5. **Express in the form of** $\mathbf{a + ib}$, **if** $\mathbf{z = \frac{(1+i)^2}{3-i}}$. Level-2(Understanding)

Solution:

Since $z = \frac{(1+i)^2}{3-i} = \frac{1^2+i^2+2.1.i}{3-i} = \frac{1-1+2i}{3-i} = \frac{2i}{3-i} = \frac{2i(3+i)}{(3-i)(3+i)} = \frac{6i+2i^2}{3^2-i^2} = \frac{6i+2(-1)}{9—1}$
$= \frac{6i-2}{9+1}$

$= \frac{-2+6i}{10} = \frac{-2}{10} + \frac{6}{10}i = \frac{-1}{5} + \frac{3}{5}i$

$\therefore a = \frac{-1}{5}$ and $b = \frac{3.}{5}$

Thus, a+ib, form of $z = \frac{(1+i)^2}{3-i}$ is $\frac{-1}{5} + \frac{3}{5}i$

6. **Express $-1+\sqrt{3i}$ in polar form.** Level-2(Understanding)

Solution:

$\mathbb{Z} = -1+\sqrt{3}i = |\mathbb{Z}|e^{i\theta}$ (where $|\mathbb{Z}| =$ modulus of the complex number and θ
$=$ amplitude of the complex number)

$|\mathbb{Z}| = \sqrt{(-1)^2 + (\sqrt{3})^2} = \sqrt{1` + 3} = \sqrt{4} = 2$

$\theta = \pi - \tan^{-1}\frac{\sqrt{3}}{1} = \pi - \tan^{-1}\sqrt{3} = \pi - \frac{\pi}{3} = \frac{2\pi}{3}$

$\therefore \mathbb{Z} = 2e^{\frac{i2\pi}{3}}$

7. **Find the value of x and y if $(x-2y)+3yi = 4-6i$.** Level-2(Understanding)

Solution:

comparing the real and imaginary parts we get ,

$x - 2y = 4, 3y = -6$

$=> 3y = -6, \quad => y = \frac{-6}{3} = -2$

now , $x - 2(-2) = 4$

$=> x + 4 = 4$

$=> x = 4 - 4 = 0$

Thus, x=0, y =-2

8. **If w is the cube root of unity find the value of $(1+w)^5$** Level-2(Understanding)

Solution:

$(1+w)^5 = (-w^2)^5$ (since $1 + w + w^2 = 0 \therefore (1+w) = -w^2$)

$\therefore (1+w)^5 = -w^{10} = -w^9.w = -(w^3)^3.w = -(1)^3.w = -1.w = -w$ (since$w^3 = 1$)

9. **Find the square root of 2i .** Level-2(Understanding)

Solution:

$\sqrt{2i} = \sqrt{1 - 1 + 2i} = \sqrt{(1)^2 + (i)^2 + 2i} = \sqrt{(1 + i)^2} = \pm (1 + i)$

10. **Find the value of $\mathbf{i^{17} + i^{20} - i^{13}}$.** Level-2(Understanding)

Solution:

$i^{17} + i^{20} - i^{13}$

$= i^{16}.i + \left(i^4\right)^5 - i^{12}.i$

$= (i^4)^4.i + (i^4)^5 - (i^4)^3.i = 1^4.i + 1^5 - 1^3.i$ (since $i^4 = 1$)

$= i + 1 - i$

$= 1$

11. **Find the multiplicative inverse of $\mathbf{2 + 3i}$.** Level-2(Understanding)

Solution:

Let $z = 2 + 3i$

Multiplicative inverse of $z = \dfrac{1}{z} = \dfrac{1}{2+3i} = \dfrac{1(2-3i)}{(2+3i)(2-3i)} = \dfrac{(2-3i)}{2^2 - (3i)^2}$

$= \dfrac{(2-3i)}{4 - 9(-1)} = \dfrac{(2-3i)}{4+9} = \dfrac{(2-3i)}{13} = \dfrac{2}{13} - \dfrac{3}{13}i$

12. **Find the modulus and amplitude of $\mathbf{\frac{1}{1-i}}$.** Level-2(Understanding)

Solution:

Let $z = \dfrac{1}{1-i} = \dfrac{1(1+i)}{(1-i)(1+i)} = \dfrac{1+i}{1^2 - i^2} = \dfrac{1+i}{1+1} = \dfrac{1+i}{2} = \dfrac{1}{2} + \dfrac{1}{2}i$

$\therefore z = \dfrac{1}{2} + \dfrac{1}{2}i$, Here $a = \dfrac{1}{2}$ and $b = \dfrac{1}{2}$

$|z| = \sqrt{\left(\dfrac{1}{2}\right)^2 + \left(\dfrac{1}{2}\right)^2} = \sqrt{\dfrac{1}{4} + \dfrac{1}{4}} = \sqrt{\dfrac{2}{4}} = \sqrt{\dfrac{1}{2}} = \dfrac{1}{\sqrt{2}}$

Amplitude $(\theta) = \tan^{-1}\left(\dfrac{b}{a}\right) = \tan^{-1}\left(\dfrac{\frac{1}{2}}{\frac{1}{2}}\right) = \tan^{-1}(1) = \dfrac{\pi}{4.}$

13. **Prove that $\mathbf{P(n, n) = P(n, n-1)}$.** Level-2(Understanding)

Proof:

$L.H.S. = P(n, n) = \dfrac{n!}{(n-n)!} = \dfrac{n!}{(0)!} = \dfrac{n!}{1} = n!$

$$\text{R.H.S.} = P(n, n-1) = \frac{n!}{(n-(n-1))!} = \frac{n!}{(n-n+1)!} = \frac{n!}{(1)!} = n!$$

$\therefore$ L.H.S. = R.H.S. (Proved)

14. **If $P(n, r) = 1680$, $C(n, r) = 70$, then find n and r.** Level-2(Understanding)

Solution:

Given that $P(n, r) = 1680, C(n, r) = 70$

$$\frac{P(n, r)}{C(n, r)} = \frac{1680}{70}$$

$$=> \frac{\frac{n!}{(n-r)!}}{\frac{n!}{r!\,(n-r)!}} = 24$$

$$=> \frac{n!}{(n-r)!} \times \frac{r!\,(n-r)!}{n!} = 24$$

$=> r! = 24 = 4!$

$=> r = 4.$

Since $P(n, r) = 1680$

$P(n, 4) = 1680$

$$=> \frac{n!}{(n-4)!} = 1680$$

$$=> \frac{n(n-1)(n-2)(n-3)(n-4)!}{(n-4)!} = 8 \times 7 \times 6 \times 5$$

$=> n(n-1)(n-2)(n-3) = 8 \times 7 \times 6 \times 5$

$=> n = 8$

15. If **$C(20, r+6) = C(20, r+2)$, then find r.** Level-2(Understanding)

Solution:

Given $C(20, r+6) = C(20, r+2)$
$=> r + 6 + r + 2 = 20$
$=> 2r + 8 = 20$
$=> 2r = 12$
$=> r = 6.$

16. **Compute $P(n, r)$ and $C(n, r)$, if $n = 10$ and $r = 3$.** Level-2(Understanding)

Solution:

$$P(n,r) = P(10,3) = \frac{10!}{(10-3)!} = \frac{10!}{7!} = \frac{10 \times 9 \times 8 \times 7!}{7!} = 720.$$
$$C(n,r) = C(10,3) = \frac{10!}{3!\,(10-3)!} = \frac{10!}{3!\,7!} = \frac{10 \times 9 \times 8 \times 7!}{(3 \times 2 \times 1)7!} = 120.$$

17. **Find the10th term of** $\left(2x^2 + \frac{1}{x}\right)^{12}$. Level-2(Understanding)

Solution:

$$t_{10} = t_{9+1} = C(12,9)\left(2x^2\right)^{12-9}\left(\frac{1}{x}\right)^9 = C(12,9)\left(2x^2\right)^3 \frac{1}{x^9} = \frac{12!}{9!\,(12-9)!} 2^3 x^6 \frac{1}{x^9} = \frac{12!}{9!\,3!} 2^3 \frac{1}{x^3}$$
$$= \frac{12 \times 11 \times 10 \times 9!}{9!\,(3 \times 2 \times 1)} 2^3 \frac{1}{x^3} = \frac{1760}{x^3}$$

18. **Find the middle term of** $\left(2x^2 - \frac{1}{x}\right)^7$. Level-2(Understanding)

Solution:

Number of terms in this expansion is 8. Hence there are two middle terms

i. e. 4th term and 5th term.

$$4\text{th term} = t_4 = t_{3+1} = (-1)^3 C(7,3)\left(2x^2\right)^{7-3}\left(\frac{1}{x}\right)^3 = (-1)\frac{7!}{3!\,(7-3)!}\left(2x^2\right)^4 \frac{1}{x^3}$$
$$= (-1)\frac{7!}{3!\,4!} 2^4 x^8 \frac{1}{x^3} = (-1)\frac{7 \times 6 \times 5 \times 4!}{(3 \times 2 \times 1)4!} 2^4 x^5 = -560x^5.$$

$$5\text{th term} = t_5 = t_{4+1} = (-1)^4 C(7,4)\left(2x^2\right)^{7-4}\left(\frac{1}{x}\right)^4 = \frac{7!}{4!\,(7-4)!}\left(2x^2\right)^3 \frac{1}{x^4} = \frac{7!}{3!\,4!} 2^3 x^6 \frac{1}{x^4}$$
$$= \frac{7 \times 6 \times 5 \times 4!}{(3 \times 2 \times 1)4!} 2^3 x^2 = 280x^2.$$

19. **Find the number of term in the expansion** $\left(x^2 - 2 + \frac{1}{x^2}\right)^6$. Level-2(Understanding)

Solution:

$$\left(x^2 - 2 + \frac{1}{x^2}\right)^6 = \left((x)^2 - 2 \times x \times \frac{1}{x} + \left(\frac{1}{x}\right)^2\right)^6 = \left[\left(x - \frac{1}{x}\right)^2\right]^6 = \left(x - \frac{1}{x}\right)^{12}$$

∴ Total number of term in this expansion is 13.

20. **In the expansion of** $(1 + x)^n$, **prove that** $C_0 + C_1 + C_2 + C_3 + \cdots + C_n = 2^n$.
Level-3(Applying)

Solution:

By Binomial theorem, we have

$$(1 + x)^n = C_0 + C_1 x + C_2 x^2 + C_3 x^3 + \cdots + C_n x^n \cdots\cdots (Eq1)$$

Putting $x = 1$, in (Eq1), we get

$(1+1)^n = C_0 + C_1 + C_2 + C_3 + \cdots\cdots + C_n$

$=> C_0 + C_1 + C_2 + C_3 + \cdots\cdots + C_n = 2^n.$

21. If $\mathbf{C(18, r) = C(18, r+2)}$, **then find** $\mathbf{C(r, 5)}$. Level-2(Understanding)

Solution:

$C(18, r) = C(18, r+2)$

$=> r + (r+2) = 18$

$=> 2r + 2 = 18$

$=> 2r = 18$-2=16

$=> r = 8$

$\therefore C(r, 5) = C(8,5) = \frac{8!}{5!\ (8-5)!} = \frac{8 \times 7 \times 6 \times 5!}{5!\ 3!} = \frac{8 \times 7 \times 6}{3 \times 2 \times 1} = 56$

22. Find n, if $\mathbf{P(n, 4) : P(n-1, 3) = 9 : 1}$. Level-3(Applying)

Solution:

Given $P(n, 4) : P(n-1, 3) = 9 : 1$

$=> \frac{P(n, 4)}{P(n-1, 3)} = \frac{9}{1}$

$=> \frac{\frac{n!}{(n-4)!}}{\frac{(n-1)!}{(n-1-3)!}} = 9$

$=> \frac{\frac{n!}{(n-4)!}}{\frac{(n-1)!}{(n-4)!}} = 9$

$=> \frac{n(n-1)!}{(n-1)!}$=9

=> n=9.

23. If $\mathbf{n}$, $\mathbf{r}$ are positive integers, such that $\mathbf{1 \le r \le n}$, then show that $\frac{\mathbf{C(n.r)}}{\mathbf{C(n, r-1)}} = \frac{\mathbf{n-r+1}}{\mathbf{r}}$

Level-3(Applying)

Solution:

L. H. S.

$\frac{C(n.r)}{C(n, r-1)} = \frac{\frac{n!}{r!\ (n-r)!}}{\frac{n!}{(r-1)!\,(n-(r-1))!}} = \frac{n!}{r!\ (n-r)!} \times \frac{(r-1)!\,(n-(r-1))!}{n!}$

$= \frac{(r-1)!\,(n-r+1)!}{r \times (r-1)!\ (n-r)!} = \frac{(n-r+1)(n-r)!}{r\,(n-r)!}$

$= \dfrac{(\mathrm{n-r+1})}{\mathrm{r}} = \mathrm{RHS(proved)}$

24. Show that $\mathbf{P(n,r) = P(n-1,r) + rP(n-1,r-1)}$. Level-3(Applying)

Solution:

R.H.S.

$$\mathrm{P}(n-1,r) + rP(n-1,r-1) = \frac{(n-1)!}{(n-1-r)!} + r\frac{(n-1)!}{(n-1-(r-1))!}$$

$$= \frac{(n-1)!}{(n-r-1)!} + r\frac{(n-1)!}{(n-1-r+1)!}$$

$$= \frac{(n-1)!}{(n-r-1)!} + r\frac{(n-1)!}{(n-r)!}$$

$$= \frac{(n-1)!}{(n-r-1)!} + r\frac{(n-1)!}{(n-r)(n-r-1)!}$$

$$= \frac{(n-1)!}{(n-r-1)!}\left[1 + \frac{r}{(n-r)}\right]$$

$$= \frac{(n-1)!}{(n-r-1)!}\left[\frac{n-r+r}{(n-r)}\right]$$

$$= \frac{(n-1)!}{(n-r-1)!}\left[\frac{n}{(n-r)}\right] = \frac{n!}{(n-r)!}$$

$$= P(n,r) = LHS(Proved)$$

25. Find the middle terms in the expansion of $\left(\frac{\boldsymbol{a}}{\boldsymbol{x}} + \frac{\boldsymbol{x}}{\boldsymbol{a}}\right)^{\mathbf{10}}$. Level-2(Understanding)

Solution:

The Required middle term $= t_{\frac{10}{2}+1}$

$$= t_6 = t_{5+1} = C(10,5)\left(\frac{a}{x}\right)^{10-5}\left(\frac{x}{a}\right)^5 = \frac{10!}{5!\,(10-5)!}\left(\frac{a}{x}\right)^5\left(\frac{x}{a}\right)^5$$

$$= \frac{10\times9\times8\times7\times6\times5!}{5!5\times4\times3\times2\times1}\frac{a^5}{x^5}\frac{x^5}{a^5} = 252.$$

26. Using Binomial Theorem, Find the value of $(\mathbf{99})^{\mathbf{4}}$. Level-3(Applying)

Solution:

$$(99)^4 = (100-1)^4$$

$$= C(4,0)100^4 - C(4,1)100^3 + C(4,2)100^2 - C(4,3)100 + C(4,4)$$

$$= 100^4 - 4\times100^3 + 6\times100^2 - 4\times100 + 1$$

$(\because C(4,0) = C(4,4) = 1, C(4,1) = C(4,3) = 4 \; and \; C(4,2) = 6\,)$

$= 100000000 - 4000000 + 60000 - 400 + 1 \;\; = 96059601.$

27. Find the co-efficient of $\boldsymbol{x}^{\mathbf{32}}$ in the expansion of $\left(\boldsymbol{x}^{\mathbf{4}} - \frac{\mathbf{1}}{\boldsymbol{x}^{\mathbf{3}}}\right)^{\mathbf{15}}$. Level-2(Understanding)

Solution:

Let (r+1)th term contains coefficient of x^{32}.

$$t_{r+1} = (-1)^r C(15,r)\left(x^4\right)^{15-r}\left(\frac{1}{x^3}\right)^r$$

$$= (-1)^r C(15,r)x^{60-4r}\frac{1}{x^{3r}}$$

$$= (-1)^r C(15,r)x^{60-4r-3r} = (-1)^r C(15,r)x^{60-7r}$$

$\therefore 60 - 7r = 32$

$=> 7r = 60 - 32 = 28.$

$=> r = 4$

$\therefore\ t_{4+1} = t_5 = (-1)^4 C(15,4) x^{60-7\times 4} = C(15,4)x^{32}$

∴The co-efficient of x^{32} is $C(15,4)$.

28. Find the 5th term in the expansion of $\left(6x - \frac{a^3}{x}\right)^{10}$. Level-2(Understanding)

Solution:

$$t_5 = t_{4+1} = (-1)^4 C(10,4)(6x)^{10-4}\left(\frac{a^3}{x}\right)^4$$

$$= C(10,4)(6x)^6 \frac{(a^3)^4}{x^4}$$

$$= C(10,4)6^6 x^6 \frac{a^{12}}{x^4} = C(10,4)6^6 a^{12} x^2.$$

29. Find the middle term in the expansion $\left(1 + 2x + x^2\right)^7$. Level-2(Understanding)

Solution:

$$\left(1 + 2x + x^2\right)^7 = \left\{(x+1)^2\right\}^7 = \left(x^2+1\right)^{14}.$$

No of terms in this expansion is 15.

8*th term is the middle term of this expansion*.

$$t_8 = t_{7+1} = C(14,7)x^7 = \frac{14!}{7!(14-7)!}x^7 = \frac{14\times13\times12\times11\times10\times9\times8\times7!}{7\times6\times5\times4\times3\times2\times1\times7!}x^7 = 3232x^7.$$

30. State Binomial theorem for positive integer index. Level-1(Remembering)

Solution:

If x& y are real numbers, then for all $n \in N$,

$$(x+y)^n = C(n,0)x^n + C(n,1)x^{n-1}y + C(n,2)x^{n-2}y^2 + \cdots\cdots + C(n,n)y^n$$

$$\text{i.e. } (x+y)^n = \sum_{r=0}^{n} C(n,r)x^{n-r}y^r,\ 0 \le r \le n.$$

Here $C(n,0), C(n,1), C(n,2), \cdots\cdots C(n,n)$ are called Binomial coefficients.

Extra Questions (2 Marks)

31. Resolve into partial fractions $\frac{2x+1}{(x-2)(x-3)}$ Level-2(Understanding)

Solution: Let $\frac{2x+1}{(x-2)(x-3)} = \frac{A}{x-2} + \frac{B}{x-3} = \frac{A(x-3)+B(x-2)}{(x-2)(x-3)}$

$\Rightarrow 2x + 1 = A(x-3) + B(x-2)$ ------ (1)

Put $x = 2$ *in both sides of equation*(1), *we get* $5 = A(-1) \Rightarrow A = -5$

Put $x = 3$ *in both sides of equation*(1), *we get* $7 = B(1) \Rightarrow B = 7$

So the required partial fraction

$$\frac{2x+1}{(x-2)(x-3)} = \frac{-5}{x-2} + \frac{7}{x-3}$$

32. Resolve into partial fractions $\frac{x}{(x+1)(x+3)}$ Level-2(Understanding)

Solution: Let $\frac{x}{(x+1)(x+3)} = \frac{A}{x+1} + \frac{B}{x+3} = \frac{A(x+3)+B(x+1)}{(x+1)(x+3)}$

$\Rightarrow x = A(x+3) + B(x+1) ------ (1)$

Put $x =-1$ $in\ both\ sides\ of\ equation(1), we\ get\ -1 = A(2) \Rightarrow A =-\frac{1}{2}$

Put $x =-3$ $in\ both\ sides\ of\ equation(1), we\ get\ -3 = B(-2) \Rightarrow B = \frac{3}{2}$

So the required partial fraction

$\frac{x}{(x+1)(x+3)} = \frac{-\frac{1}{2}}{x+1} + \frac{\frac{3}{2}}{x+3} =-\frac{1}{2(x+1)} + \frac{3}{2(x+3)}$

33. Resolve into partial fractions $\frac{1}{x^2-1}$ Level-2(Understanding)

Solution: Let $\frac{1}{x^2-1} = \frac{1}{(x-1)(x+1)} = \frac{A}{x-1} + \frac{B}{x+1} = \frac{A(x+1)+B(x-1)}{(x-1)(x+1)}$

$\Rightarrow 1 = A(x+1) + B(x-1) ------ (1)$

Put $x = 1$ $in\ both\ sides\ of\ equation(1), we\ get\ 1 = A(2) \Rightarrow A = \frac{1}{2}$

Put $x =-1$ $in\ both\ sides\ of\ equation(1), we\ get\ 1 = B(-2) \Rightarrow B =-\frac{1}{2}$

So the required partial fraction

$\frac{1}{x^2-1} = \frac{\frac{1}{2}}{x-1} + \frac{-\frac{1}{2}}{x+1} = \frac{1}{2(x-1)} - \frac{1}{2(x+1)}$

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5 Marks Questions & Solutions ***Taxonomy Level***

1. If $1, \omega, \omega^2$ are cube roots of unity prove that $(1-\omega+\omega^2)^5 + (1+\omega-\omega^2)^5 = 32$

Level-3(Applying)

Solution:

L. H. S.

$(1-\omega+\omega^2)^5 + (1+\omega-\omega^2)^5$

$= (1+\omega^2-\omega)^5 + (1+\omega-\omega^2)^5$

$= (-\omega-\omega)^5 + (-\omega^2-\omega^2)^5$ $(\because \text{since } 1+\omega+\omega^2 = 0 \therefore 1+\omega^2 =-\omega \text{ and } 1+\omega =-\omega^2)$

$= (-2\omega)^5 + (-2\omega^2)^5$

$= -32\omega^5 - 32\omega^{10}$

$=-32\omega^3 \cdot \omega^2 - 32\omega^9 \cdot \omega$

$=-32(\omega^3)\omega^2 - 32(\omega^3)^3 . \omega$

$=-32(1)\omega^2 - 32(1)^3 \cdot \omega$ $(\because \text{since} \omega^3 = 1)$

$=-32\omega^2 - 32\omega$

$= -32(\omega^2 + \omega)$

$= -32(-1)$ ($\therefore$ since $1 + \omega + \omega^2 = 0 \therefore \omega^2 + \omega = -1$)

$= 32 = $ (R. H. S)(Proved)

2. **If $x + \frac{1}{x} = 2\cos\theta$, then prove that $x^n + \frac{1}{x^n} = 2\cos n\theta$ and $x^n - \frac{1}{x^n} = 2i\sin(n\theta)$**

Level-3(Applying)

Solution:

Given , $x + \frac{1}{x} = 2\cos\theta$

$=> \frac{x^2+1}{x} = 2\cos\theta$

$=> x^2 + 1 = 2x\cos\theta$

$=> x^2 - 2x\cos\theta + 1 = 0$

$=> x = \frac{-(-2\cos\theta) \pm \sqrt{(-2\cos\theta)^2 - 4(1)(1)}}{2(1)}$

$=> x = \frac{2\cos\theta \pm \sqrt{4\cos^2\theta - 4}}{2}$

$=> x = \frac{2\cos\theta \pm \sqrt{-4(-\cos^2\theta + 1)}}{2}$

$=> x = \frac{2\cos\theta \pm \sqrt{-4\sin^2\theta}}{2}$

$=> x = \frac{2\cos\theta \pm 2i\sin\theta}{2}$

$=> x = \cos\theta \pm i\sin\theta$

take, $x = \cos\theta + i\sin\theta$

$=> x^n = \cos n\theta + i\sin n\theta$(by De Moivre theorem)

$=> \frac{1}{x^n} = \frac{1}{\cos n\theta + i\sin n\theta} = (\cos n\theta + i\sin n\theta)^{-1}$

$=> \frac{1}{x^n} = \cos(-1)(n\theta) + i\sin(-1)(n\theta)$ (by De Moivre's theorem)

$=> \frac{1}{x^n} = \cos(-n\theta) + i\sin(-n\theta)$

$=> \frac{1}{x^n} = \cos(n\theta) - i\sin(n\theta)$

$\therefore x^n + \frac{1}{x^n} = \cos n\theta + i\sin n\theta + \cos(n\theta) - i\sin(n\theta) = 2\cos n\theta$

$x^n - \frac{1}{x^n} = (\cos n\theta + i\sin n\theta) - (\cos(n\theta) - i\sin(n\theta))$

$$x^n - \frac{1}{x^n} = \cos n\theta + i \sin n\theta - \cos n\theta + i \sin(n\theta) = 2i \sin n\theta$$

3. **Resolve into pratial fractions**: $\frac{x^2+x-2}{x(x+3)(x-2)}$ **level-2(Understanding)**

Solution:

Let $\frac{x^2+x-2}{x(x+3)(x-2)} = \frac{A}{x} + \frac{B}{x+3} + \frac{C}{x-2}$

$=> \frac{x^2+x-2}{x(x+3)(x-2)} = \frac{A(x+3)(x-2) + Bx(x-2) + Cx(x+3)}{x(x+3)(x-2)}$

$=> x^2 + x - 2 = A(x+3)(x-2) + Bx(x-2) + Cx(x+3)$

put $x = 0$ $=> -2 = A(0+3)(0-2)$

$=> -2 = A(3)(-2)$

$=> -2 = -6A$

$=> A = \frac{-2}{-6} = \frac{1}{3}$

Put $x = 2$ $=> 2^2 + 2 - 2 = C(2)(2+3)$

$=> 4 + 2 - 2 = C(2)(5)$

$=> 4 = 10C$

$=> C = \frac{4}{10} = \frac{2}{5}$

Put $x = -3$ $=> (-3)^2 - 3 - 2 = B(-3)(-3-2)$

$=> 9 - 3 - 2 = B(-3)(-5)$

$=> 4 = 15B$

$=> B = \frac{4}{15}$

$\therefore \frac{x^2+x-2}{x(x+3)(x-2)} = \frac{\frac{1}{3}}{x} + \frac{\frac{4}{15}}{x+3} + \frac{\frac{2}{5}}{x-2}$

$=> \frac{x^2+x-2}{x(x+3)(x-2)} = \frac{1}{3x} + \frac{4}{15(x+3)} + \frac{2}{5(x-2)}$

4. **Resolve into pratial fractions**: $\frac{1+2x}{(x+2)^2(x-1)^2}$ **level-2(Understanding)**

Solution:

Let $\frac{1+2x}{(x+2)^2(x-1)^2} = \frac{A}{x+2} + \frac{B}{(x+2)^2} + \frac{C}{x-1} + \frac{D}{(x-1)^2}$

$$=> \frac{1+2x}{(x+2)^2(x-1)^2} = \frac{A(x+2)(x-1)^2 + B(x-1)^2 + C(x+2)^2(x-1) + D(x+2)^2}{(x+2)^2(x-1)^2}$$

$$=> 1+2x = A(x+2)(x-1)^2 + B(x-1)^2 + C(x+2)^2(x-1) + D(x+2)^2$$

Substiute $x - 1 = 0$ i.e. $x = 1$

$$=> 3 = D(3)^2$$

$$=> 3 = D(9)$$

$$=> D = \frac{3}{9} = \frac{1}{3}$$

Again substitute $x + 2 = 0$ i.e. $x =- 2$

$$=> 1 - 4 = B(-3)^2$$

$$=>- 3 = B(3)^2$$

$$=> B = \frac{-3}{9} = \frac{-1}{3}$$

Equating the coefficient of x^3 and x^2 on both sides, we get

$$=> A + C = 0$$

$$=> A =- C$$

And $B + 3C + D = 0$

$$=> \frac{-1}{3} + 3C + \frac{1}{3} = 0$$

$$=> 3C = 0$$

$$=> C = 0$$

$$=> A =- C = 0$$

So our required partial fraction is

$$\frac{1+2x}{(x+2)^2(x-1)^2} = \frac{0}{x+2} + \frac{\frac{-1}{3}}{(x+2)^2} + \frac{0}{x-1} + \frac{\frac{1}{3}}{(x-1)^2}$$

$$\therefore \frac{1+2x}{(x+2)^2(x-1)^2} = \frac{-1}{3(x+2)^2} + \frac{1}{3(x-1)^2}$$

5. **Resolve into pratial fractions**: $\frac{x^2+6}{(x^2+1)(x^2+4)}$ **Level-2(Understanding)**

Solution:

Let $\frac{x^2+6}{(x^2+1)(x^2+4)} = \frac{Ax+B}{(x^2+1)} + \frac{Cx+D}{(x^2+4)}$

$$=> \frac{x^2+6}{(x^2+1)(x^2+4)} = \frac{(Ax+B)(x^2+4) + (Cx+D)(x^2+1)}{(x^2+4)(x^2+1)}$$

$$=> x^2 + 6 = (Ax+B)(x^2+4) + (Cx+D)(x^2+1).$$

Equating the coefficient of x^3, x^2, x and the constant terms on both sides, we get

Cofeeicient of x^3 on both side $=> 0 = A + C ------ Eq(1)$

Cofeeicient of x^2 on both side $=> 1 = B + D ------ Eq(2)$

Cofeeicient of x on both side $=> 0 = 4A + C ------ Eq(3)$

Constant term on both side $=> 6 = 4B + D ------ Eq(4)$

From Eq(1) and Eq(3)

We have $A = 0$ and $C = 0$

From Eq(2) and Eq(4)

We have $B = \frac{5}{3}$ and $D = \frac{-2}{3}$

So our required partial fraction is

$$\frac{x^2+6}{(x^2+1)(x^2+4)} = \frac{0x+\frac{5}{3}}{(x^2+1)} + \frac{0x+\frac{-2}{3}}{(x^2+4)}$$

$$\therefore \frac{x^2+6}{(x^2+1)(x^2+4)} = \frac{5}{3(x^2+1)} + \frac{-2}{3(x^2+4)}$$

6. **If $P(n,4) = 2P(5,3)$, then find the value of n.** **Level-3(Applying)**

Solution:

Given that $P(n,4) = 2P(5,3)$,

$$=> \frac{n!}{(n-4)!} = 2\frac{5!}{(5-3)!}$$

$$=> \frac{n(n-1)(n-2)(n-3)(n-4)!}{(n-4)!} = 2\frac{5\times4\times3\times2\times1}{2\times1}$$

$$=> n(n-1)(n-2)(n-3) = 120$$

$$=> n(n-3)(n-1)(n-2) = 120$$

$$=> (n^2-3n)(n^2-3n+2) = 120$$

Substitute $n^2 - 3n = m$, we get

$$=> m(m+2) = 120$$

$$=> m^2 + 2m - 120 = 0$$

$$=> m^2 + 12m - 10m - 120 = 0$$

$$=> m(m+12) - 10(m+12) = 0$$

$$=> (m+12)(m-10) = 0$$

$$=> (m+12) = 0 \text{ or } (m-10) = 0$$

$$=> m = -12 \text{ or } m = 10$$

again substitutre the value of m, we get

$=> n^2 - 3n =- 12 \text{ or } n^2 - 3n = 10$

$=> n^2 - 3n + 12 = 0 \text{ or } n^2 - 3n - 10 = 0$

$$=> n = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(1)(12)}}{2(1)} \quad \text{or } n = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(1)(-10)}}{2(1)}$$

$$=> n = \frac{3 \pm \sqrt{9 - 48}}{2} \quad \text{or } n = \frac{3 \pm \sqrt{9 + 40}}{2}$$

$$=> n = \frac{3 \pm \sqrt{-39}}{2} \quad \text{or } n = \frac{3 \pm \sqrt{49}}{2}$$

$$=> n = \frac{3 \pm \sqrt{39}i}{2} \quad \text{or } n = \frac{3 \pm 7}{2}$$

$$=> n = \frac{3 + \sqrt{39}i}{2} \text{ and } \frac{3 - \sqrt{39}i}{2} \quad \text{or } n = \frac{3 + 7}{2} \text{ and } \frac{3 - 7}{2}$$

$$=> n = \frac{3 + \sqrt{39}i}{2} \text{ and } \frac{3 - \sqrt{39}i}{2} \quad \text{or } n = 5 \text{ and } -2$$

$=> n = 5$ (*negelcting the* $-$ *ve value and the imaginary value*).

7. If $1 \leq r \leq n$, then prove that $C(n, r) + C(n, r + 1) = C(n + 1, r + 1)$. **Level-3(Applying)**

Proof:

<u>L.H.S.</u>

$$C(n,r) + C(n,r+1) \equiv \frac{n!}{r!\,(n-r)!} \pm \frac{n!}{(r+1)!\,(n-(r+1))!}$$

$$= \frac{n!}{r!\,(n-r) \times (n-r-1)!} + \frac{n!}{(r+1) \times r!\,(n-r-1)!}$$

$$= \frac{n!}{r! \times (n-r-1)!}\left[\frac{1}{n-r} + \frac{1}{(r+1)}\right]$$

$$= \frac{n!}{r! \times (n-r-1)!}\left[\frac{r+1+n-r}{(n-r)(r+1)}\right]$$

$$= \frac{n!}{r! \times (n-r-1)!}\left[\frac{n+1}{(n-r)(r+1)}\right]$$

$$= \frac{n!\,(n+1)}{r! \times (n-r-1)! \times (n-r) \times (r+1)}$$

$$= \frac{(n+1)!}{(n-r)! \times (r+1)!}$$

$$= \frac{(n+1)!}{(r+1)! \times ((n+1)-(r+1))!}$$

$$= C(n+1, r+1) = R.H.S.\,(\text{Proved})$$

8. Find the term independent of x in the expansion $\left(\frac{3}{2}x^2 - \frac{1}{3x}\right)^9$. **Level-2(Understanding)**

Solution:

Let $t_{(r+1)}$th term is independent of x.

$$\text{But } t_{(r+1)} = (-1)^r C(9,r)\left(\frac{3}{2}x^2\right)^{9-r}\left(\frac{1}{3x}\right)^r$$

$$= (-1)^r C(9,r)\left(\frac{3}{2}\right)^{9-r}(x^2)^{9-r}\left(\frac{1}{3}\right)^r\left(\frac{1}{x}\right)^r$$

$$= (-1)^r C(9,r)\frac{3^{9-r}}{2^{9-r}}x^{18-2r}\frac{1}{3^r}\frac{1}{x^r}^r$$

$$= (-1)^r C(9,r)\frac{3^{9-2r}}{2^{9-r}}x^{18-3r}$$

Since this term is independent of x,

$=> 18 - 3r = 0$

$=> 18 = 3r$

$=> r = 6.$

$$\therefore t_{(r+1)} = (-1)^r C(9,r)\frac{3^{9-2r}}{2^{9-r}}x^{18-3r}$$

$$t_{(6+1)} = (-1)^6 C(9,6)\frac{3^{9-2\times 6}}{2^{9-6}}x^{18-3\times 6}$$

$$t_7 = \frac{9!}{6!\,(9-6)!}\frac{3^{9-12}}{2^3}x^0$$

$$t_7 = \frac{9\times 8\times 7\times 6!}{6!\,3!}\frac{3^{-3}}{2^3} = \frac{7}{18}$$

Hence the 7th term is independent of x and the term is $\frac{7}{18}$.

9. Find the Square root of $1 + 4\sqrt{3}i$. **Level-2(Understanding)**

Solution:

Let $\sqrt{1+4\sqrt{3}i} = x + iy$

Squaring both side , we get

$$\left(\sqrt{1+4\sqrt{3}i}\right)^2 = (x+iy)^2$$

$=> 1 + 4\sqrt{3}i = x^2 + (iy)^2 + 2\times x\times iy$

$=> 1 + 4\sqrt{3}i = x^2 - y^2 + 2\times x\times iy$

Equating the real part and imaginary part on both sides, we get

$x^2 - y^2 = 1$ ------ Eq(1)

$2xy = 4\sqrt{3}$ ------ Eq(2)

$\Rightarrow (x^2 + y^2)^2 = (x^2 - y^2)^2 + 4x^2y^2$

$\Rightarrow (x^2 + y^2)^2 = (x^2 - y^2)^2 + (2xy)^2$

$\Rightarrow (x^2 + y^2)^2 = (1)^2 + \left(4\sqrt{3}\right)^2$

$\Rightarrow (x^2 + y^2)^2 = 1 + 48$

$\Rightarrow (x^2 + y^2)^2 = 49$

$\Rightarrow x^2 + y^2 = 7$ ------- Eq(3)

Solving Eq(1) and Eq(3) , we get

$x = \pm 2$ and $y = \pm \sqrt{3}$

Hence square root of $1 + 4\sqrt{3}i$ i.e.

$\therefore \sqrt{1 + 4\sqrt{3}i} = \pm \left(2 + \sqrt{3}i\right).$

10. Find the coefficient of $\frac{1}{y^{10}}$ in the expansion $\left(y^3 + \frac{a^7}{y^5}\right)^{10}$. **Level-2(Understanding)**

Solution.

Let $(r + 1)$th tern contains $\frac{1}{y^{10}}$

$\Rightarrow t_{(r+1)} = C(10, r)(y^3)^{10-r}\left(\frac{a^7}{y^5}\right)^r$

$= C(10, r)y^{30-3r}\ \frac{a^{7r}}{y^{5r}}$

$= C(10, r)\frac{a^{7r}}{y^{5r-(30-3r)}}$

$= C(10, r)\frac{a^{7r}}{y^{5r-30+3r}}$

$= C(10, r)\frac{a^{7r}}{y^{8r-30}}$

But $y^{8r-30} = y^{10}$

$\Rightarrow 8r - 30 = 10$

$\Rightarrow 8r = 30 + 10 = 40$

$\Rightarrow r = \frac{40}{8} = 5$

So,

$$=> t_{(r+1)} = C(10,r)\frac{a^{7r}}{y^{8r-30}}$$

$$=> t_{(5+1)} = C(10,5)\frac{a^{7\times 5}}{y^{8\times 5-30}}$$

$$=> t_6 = C(10,5)\frac{a^{35}}{y^{10}} = \frac{10!}{5!\,(10-5)!}\frac{a^{35}}{y^{10}}$$

$$= \frac{10\times 9\times 8\times 7\times 6\times 5!}{5!\,5!}\frac{a^{35}}{y^{10}}$$

$$= \frac{10\times 9\times 8\times 7\times 6}{5\times 4\times 3\times 2\times 1}\times a^{35}\times\frac{1}{y^{10}} = 252\times a^{35}\times\frac{1}{y^{10}}$$

Hence the 6th term contains $\frac{1}{y^{10}}$ whose co efficient is $252a^{35}$.

Extra question (5 Marks)

11. Resolve into Partial fraction : $\frac{x^2+1}{x^2-5x+6}$ **Level-2(Understanding)**

Solution: :$\frac{x^2+1}{x^2-5x+6} = 1 + \frac{5x-5}{x^2-5x+6}$

$\frac{5x-5}{x^2-5x+6}$=$\frac{5x-5}{x^2-3x-2x+6} = \frac{5x-5}{(x-3)(x-2)}$

Let $\frac{5x-5}{(x-3)(x-2)}$ = $\frac{A}{x-3} + \frac{B}{x-2}$ = $\frac{A(x-2)+B(x-3)}{(x-3)(x-2)}$

$\Rightarrow 5x - 5 = A(x-2) + B(x-3)$

Putting x=2 ,we get B = -5 and putting x = 3 ,we get A = 10

Hence $\frac{x^2+1}{x^2-5x+6} = 1 + \frac{5x-5}{x^2-5x+6}$ = $1 + \frac{10}{x-3} - \frac{5}{x-2}$

12. Expand $(1 + 2x + x^2)^3$ by using Binomial Theorem . **Level-2(Understanding)**

Solution: $(1 + 2x + x^2)^3$ = $((1 + x)^2)^3 = (1 + x)^6$

= $C(6,0) + C(6,1)x + C(6,2)x^2 + C(6,3)x^3 + C(6,4)x^4 + C(6,5)x^5 + C(6,6)x^6$

= $1 + 6x + 15x^2 + 20x^3 + 15x^4 + 6x^5 + x^6$.

13. Find the term independent of x in the expansion of $\left(\frac{x^2}{3} - \frac{4}{x^2}\right)^6$ **Level-2(Understanding)**

Solution: Term independent of x in $\left(\frac{x^2}{3}-\frac{4}{x^2}\right)^6$

Let the term independent of x be $t_{r+1} = C(n,r)\left(\frac{x^2}{3}\right)^{6-r}\left(\frac{-4}{x^2}\right)^r$

$= C(6,r)\frac{x^{12-2r}}{3^{6-r}}(-1)^r 4^r x^{-2r} = C(6,r)(-1)^r\frac{1}{3^{6-r}}4^r x^{12-2r-2r}$

$= C(6,r)(-1)^r\frac{1}{3^{6-r}}4^r x^{12-4r}$

12- 4r = 0

$\Rightarrow$ 4r = 12

$\Rightarrow$ r = 3

The term is $t_4 = C(6,3)(-1)^3\frac{1}{3^{6-3}}4^3 =- 20 \times \frac{64}{27} = \frac{-1280}{27}$

14. If α and β are the two roots of the equation $x^2-2x+4=0$, then show that $\alpha^n+\beta^n = 2^{n+1}\cos\frac{n\pi}{3}$. Level-3(Applying)

Solution:

The roots of the equation $x^2-2x+4=0$ are given by , $x = \frac{2\pm\sqrt{4-16}}{2} = \frac{2\pm 2\sqrt{3}i}{2} = 1\pm\sqrt{3}i$

Let $\alpha = 1+\sqrt{3}i$ and $\beta = 1-\sqrt{3}i$

Now, changing α and β to modulus argument form, we put

1=r cos θ and $\sqrt{3}$ =rsin θ

$r^2(\cos^2\theta+\sin^2\theta)$=1+3=4=> $r=2$

Now, $\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}} = \sqrt{3} \Rightarrow \theta = \frac{\pi}{3}$

$\alpha=2\left(\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}\right)$ and $\beta=2\left(\cos\frac{\pi}{3}-i\sin\frac{\pi}{3}\right)$

Now, $\alpha^n+\beta^n = 2^n\left[\left(\cos\frac{n\pi}{3}+i\sin\frac{n\pi}{3}\right)+\left(\cos\frac{n\pi}{3}-i\sin\frac{n\pi}{3}\right)\right]$

$=2^n 2\cos\frac{n\pi}{3} = 2^{n+1}\cos\frac{n\pi}{3}$. ■